Einstein-Podolsky-Rosen sideband entanglement in broadband squeezed light

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In this paper we investigate the squeezing of a finite-bandwidth optical field which may be thought of as originating from Einstein-Podolsky-Rosen- (EPR-) like quantum correlations between the upper and lower sidebands. Ordinarily, when EPR pairs are created from such a resource, the bandwidth is preserved, and the signal-to-noise ratio of the quadrature correlations is reduced. Here we show that by using frequency-dependent beam splitters we may create N EPR pairs from a single finite-bandwidth squeezed beam with no reduction in the correlations's signal-to-noise ratio. However, the bandwidth is reduced by a factor of N relative to the original squeezed beam. Because this transformation is linear it may be inverted to retrieve the single squeezed beam from the corresponding N EPR pairs.

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In the developing area of quantum information science nonlocal quantum entanglement plays a pivotal role [1,2]. Recently, both theoretical and experimental investigations have increasingly focused on quantum states of continuous variables in an infinite-dimensional Hilbert space. These studies typically rely on continuous-variable Einstein-Podolsky-Rosen (EPR) entangled states of light that can be efficiently generated using squeezed light and beam splitters. Such EPR states have been successfully employed, for example, in demonstrating unconditional quantum teleportation [3]. In addition, bright EPR beams have been generated experimentally by means of a nondegenerate optical parametric amplifier [4,5] and the Kerr nonlinearity of an optical fiber [6]. Entanglement produced by a single-mode squeezed state and distributed among N parties has been examined in terms of the fidelity of quantum teleportation of coherent states [7]. In Ref. [8], two equally squeezed states incident on a beam splitter, a so-called dual squeezed input (DSI), yield a bipartite entangled state with equal strength correlations between quadratures. In contrast, entanglement created at a beam splitter from a single squeezed state, so-called single-squeezed-input (SSI) entanglement, introduces correlation biases among the quadratures [8]. If maximal EPR states are defined by the minimum resources (photons) necessary to produce a particular level of entanglement [8], only DSI entanglement produces a maximal EPR state.

Now an empty optical cavity can be used as a frequency-dependent mirror or beam splitter. The simplest case is that of the mode cleaner, obtained by transmitting light resonantly through a high-finesse cavity [9]. If nonresonant light is reflected from a cavity, then the upper and lower sidebands suffer different phase shifts relative to the carrier. This can be used to rotate the phase and amplitude quadratures of an input field and has been applied in tomography of bright-light fields [10,11]. Reference [12] compared the SSI and DSI entanglement generated at an ordinary beam splitter with that generated at an empty optical cavity modeling a frequency-dependent beam splitter.

In this paper we analyze a single-mode squeezed optical field with EPR-like quantum correlations between upper and lower sidebands. We adopt the empty optical cavity as a model of a frequency-dependent beam splitter to spatially separate the upper and lower sidebands of a single beam. By repeated use of such frequency-dependent beam splitters, one squeezed light beam can be converted into N entangled DSI EPR pairs. While the strength of the correlations is not reduced in signal-to-noise ratio, the entanglement bandwidth of EPR pairs is only $1/2N$ of the squeezed input light bandwidth. In this sense the entangled pairs produced by the empty optical cavity in our scheme are distinct from those discussed in Ref. [12]. (Conversely, the N frequency-nondegenerate EPR pairs may be converted into a single-mode squeezed light beam, with overall bandwidth $2N$ times that of the individual EPR pairs.)

Consider the balanced homodyne detection of a single-mode broadband amplitude-quadrature (or phase-quadrature) squeezed beam with central frequency ω_0 and squeezing bandwidth $B=\Omega_1-\Omega_0$ as shown in Fig. 1. At the 50% beam splitter, the signal field, with annihilation operator $\hat{a}$, spatially overlaps a local oscillator (LO) $\hat{a}_{\mathrm{LO}}$ of frequency ω_0. The beam splitter's output may be expressed in terms of its input via

$$\hat{c}(t)=[\hat{a}(t)+e^{i\theta}\hat{a}_{\mathrm{LO}}(t)]/\sqrt{2}, \tag{1}$$

$$\hat{d}(t)=[\hat{a}(t)-e^{i\theta}\hat{a}_{\mathrm{LO}}(t)]/\sqrt{2},$$

where the angle θ depends on the relative phase between the signal field and the local oscillator. The difference between output photocurrents from detectors D_1 and D_2 (see Fig. 1) is written as

$$\hat{I}_{-}(t)=\hat{c}^{\dagger}(t)\hat{c}(t)-\hat{d}^{\dagger}(t)\hat{d}(t)$$

$$=\frac{1}{2}[e^{i\theta}\hat{a}^{\dagger}(t)\hat{a}_{\mathrm{LO}}(t)+e^{-i\theta}\hat{a}_{\mathrm{LO}}^{\dagger}(t)\hat{a}(t)]. \tag{2}$$

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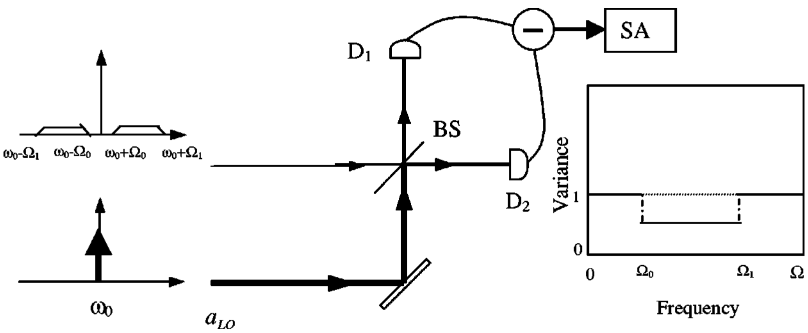

FIG. 1. Schematic of balanced homodyne detection for broadband squeezed light. Here SA is a spectrum analyzer and BS a beam splitter. The detectors are D_1 and D_2.

For the usual experimental optical homodyne scheme the condition $|\langle \hat{a}_{\rm LO}\rangle| \gg |\langle \hat{a}\rangle|$ is assumed satisfied, in which case the output photocurrent analyzed at radio frequency Ω is given by

$$\begin{aligned}\hat{I}_-(\Omega) &= \frac{1}{2}\bar{a}_{\rm LO}(\omega_0)[e^{-i\theta}\hat{a}(\omega_0-\Omega)+e^{i\theta}\hat{a}^\dagger(\omega_0+\Omega)]\\ &= \frac{1}{2}\bar{a}_{\rm LO}(\omega_0)\{e^{-i\theta}[\hat{X}(\omega_0-\Omega)+i\hat{Y}(\omega_0-\Omega)]\\ &\quad + e^{i\theta}[\hat{X}(\omega_0+\Omega)-i\hat{Y}(\omega_0+\Omega)]\}. \quad (3)\end{aligned}$$

The quadrature component of the measured signal field is therefore

$$\hat{X}(\theta,t)=[\hat{a}(t)e^{-i\theta}+\hat{a}^\dagger(t)e^{i\theta}]/\sqrt{2}, \quad (4)$$

where $\hat{X}(\theta,t)$ may be Fourier transformed into

$$\hat{X}(\theta,\Omega)=[\hat{a}(\omega_0-\Omega)e^{-i\theta}+\hat{a}^\dagger(\omega_0+\Omega)e^{i\theta}]/\sqrt{2}. \quad (5)$$

From Eqs. (3) and (5) it is obvious that the difference photocurrent is directly proportional to the quadrature component of the signal field. When $\theta=0$, the amplitude quadrature of the input field is detected:

$$\begin{aligned}\hat{X}(\Omega) &= [\hat{X}(\omega_0-\Omega)+i\hat{Y}(\omega_0-\Omega)+\hat{X}(\omega_0+\Omega)\\ &\quad -i\hat{Y}(\omega_0+\Omega)]/2, \quad (6)\end{aligned}$$

$$\langle \Delta^2\hat{X}(\Omega)\rangle = e^{-2r}<1, \quad \Omega\in[\Omega_0,\Omega_1],$$

where $\langle \Delta^2\hat{X}\rangle=\langle \hat{X}^2\rangle-\langle \hat{X}\rangle^2$ is the variance of operator $\hat{X}$, and without loss of generality we have assumed a pure squeezed signal with squeezing parameter $r>0$. When $\theta=\pi/2$, the phase quadrature of the input field is measured:

$$\begin{aligned}\hat{Y}(\Omega) &= [-i\hat{X}(\omega_0-\Omega)+\hat{Y}(\omega_0-\Omega)+i\hat{X}(\omega_0+\Omega)\\ &\quad +\hat{Y}(\omega_0+\Omega)]/2, \quad (7)\end{aligned}$$

$$\langle \Delta^2\hat{Y}(\Omega)\rangle = e^{2r}>1, \quad \Omega\in[\Omega_0,\Omega_1].$$

Here $\hat{X}(\omega_0+\Omega)$ and $\hat{Y}(\omega_0+\Omega)$ are the amplitude and phase quadratures associated with the upper-sideband mode $\hat{b}_1$, whereas $\hat{X}(\omega_0-\Omega)$ and $\hat{Y}(\omega_0-\Omega)$ are the quadratures associated with the lower-sideband mode $\hat{b}_2$. Naturally, these modes have the usual commutation relations

$$[\hat{b}_k,\hat{b}_{k'}]=[\hat{b}_k^\dagger,\hat{b}_{k'}^\dagger]=0, \ [\hat{b}_k,\hat{b}_{k'}^\dagger]=\delta_{kk'}, \quad k,k'=1,2. \quad (8)$$

Leaving the continuous-spectrum notation behind we write the quadrature-phase amplitude of these two modes as

$$\hat{X}_{b_k}=\hat{b}_k+\hat{b}_k^\dagger, \quad \hat{Y}_{b_k}=-i(\hat{b}_k-\hat{b}_k^\dagger). \quad (9)$$

Similarly, the quadrature-phase amplitudes obey the usual commutation relations

$$[\hat{X}_{b_k},\hat{X}_{b_{k'}}]=[\hat{Y}_{b_k},\hat{Y}_{b_{k'}}]=0, \quad [\hat{X}_{b_k},\hat{Y}_{b_{k'}}]=2i\delta_{kk'},$$
$$k,k'=1,2. \quad (10)$$

Performing a beam-splitting unitary transformation on the modes $\hat{b}_1$ and $\hat{b}_2$, we have

$$\hat{c}_1=\frac{1}{\sqrt{2}}(\hat{b}_1-\hat{b}_2), \quad (11)$$

$$\hat{c}_2=\frac{1}{\sqrt{2}}(\hat{b}_1+\hat{b}_2).$$

Thus the operators $\hat{c}_1$and $\hat{c}_2$ and their associated quadrature-phase amplitudes $\hat{X}_{c_1},\hat{Y}_{c_1}$ and $\hat{X}_{c_2},\hat{Y}_{c_2}$ satisfy analogous commutation relations to those just described.

According to these commutation relations, the corresponding uncertainty principles of the quadrature-phase amplitudes of the operators $\hat{c}_1$ and $\hat{c}_2$ are

$$\langle \Delta^2\hat{X}_{c_1}\rangle\langle \Delta^2\hat{Y}_{c_1}\rangle \geq 1, \quad (12)$$

$$\langle \Delta^2\hat{X}_{c_2}\rangle\langle \Delta^2\hat{Y}_{c_2}\rangle \geq 1.$$

These inequalities imply that the amplitude and phase quadratures $\hat{X}_{c_k}$ and $\hat{Y}_{c_k}$ cannot be measured simultaneously with arbitrarily high accuracy. However, the sum of $\hat{X}_{c_2}$ and $\hat{Y}_{c_2}$ and the difference of $\hat{X}_{c_1}$ and $\hat{Y}_{c_1}$ may be so measured

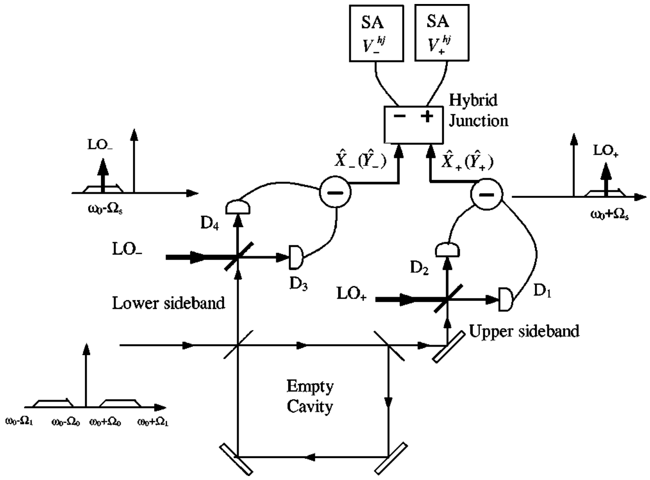

FIG. 2. Schematic illustration of EPR entanglement between the upper and lower sidebands of a single-mode squeezing using the empty cavity as the frequency-dependent beam splitter. The lower sideband is reflected off the cavity and the upper sideband is transmitted through the cavity.

[13]. The corresponding uncertainty relations for the upper and lower sidebands of the quadrature-phase amplitudes are given by

$$\langle \Delta^2 \hat{X}(\omega_0+\Omega)\rangle \langle \Delta^2 \hat{Y}(\omega_0+\Omega)\rangle \geqslant 1, \tag{13}$$
$$\langle \Delta^2 \hat{X}(\omega_0-\Omega)\rangle \langle \Delta^2 \hat{Y}(\omega_0-\Omega)\rangle \geqslant 1.$$

Thus, from Eqs. (6), (12), and (13), the upper and lower sidebands possess EPR-like entanglement correlations with quadrature-amplitude anticorrelation and quadrature-phase correlation [5]:

$$\langle \Delta^2 \hat{X}(\omega_0+\Omega)\rangle = \langle \Delta^2 \hat{X}(\omega_0-\Omega)\rangle = \langle \Delta^2 \hat{Y}(\omega_0+\Omega)\rangle = \langle \Delta^2 \hat{Y}(\omega_0-\Omega)\rangle = \frac{e^{2r}+e^{-2r}}{2} > 1, \tag{14}$$

and

$$\begin{aligned} &\langle \Delta^2[\hat{X}(\omega_0+\Omega)+\hat{X}(\omega_0-\Omega)]\rangle/2 \\ &= \langle \Delta^2[\hat{Y}(\omega_0+\Omega)-\hat{Y}(\omega_0-\Omega)]\rangle/2 \\ &= e^{-2r} < 1. \end{aligned} \tag{15}$$

At the same time, the phase-quadrature component $\langle \Delta^2 \hat{Y}(\Omega)\rangle$ must be larger than the standard quantum limit (SQL) since

$$\begin{aligned} &\langle \Delta^2[\hat{X}(\omega_0+\Omega)-\hat{X}(\omega_0-\Omega)]\rangle/2 \\ &= \langle \Delta^2[\hat{Y}(\omega_0+\Omega)+\hat{Y}(\omega_0-\Omega)]\rangle/2 \\ &= e^{2r} > 1. \end{aligned} \tag{16}$$

In order to convert these EPR-like correlations into real spatially separated EPR pairs, we shall use a frequency-dependent beam splitter. In particular, we use an optical cavity to separate the upper and lower sidebands of a single beam into spatially separated beams as shown in Fig. 2. Here we consider the use of a double-ended empty cavity as an ideal optical bandpass filter, which transmits the upper sideband resonantly from $\omega_0+\Omega_0$ to $\omega_0+\Omega_1$ and reflects the lower sideband. Thus, the upper and lower sidebands of a single beam are separated into two spatially separated beams.

These separated beams are now sent into a pair of homodyne detectors with LO frequencies of $\omega_0+\Omega_s$ (LO_+) and $\omega_0-\Omega_s$ (LO_-) to detect the upper and lower sidebands, respectively [here $\Omega_s \equiv (\Omega_0+\Omega_1)/2$]. When $\theta=0$, the amplitude quadratures of the upper and lower sidebands are detected, namely,

$$\begin{aligned} \hat{X}_+(\Omega) = [&\hat{X}(\omega_0+\Omega_s-\Omega)+i\hat{Y}(\omega_0+\Omega_s-\Omega) \\ &+\hat{X}(\omega_0+\Omega_s+\Omega)-i\hat{Y}(\omega_0+\Omega_s+\Omega)]/2, \end{aligned} \tag{17}$$

$$\begin{aligned} \hat{X}_-(\Omega) = [&\hat{X}(\omega_0-\Omega_s-\Omega)+i\hat{Y}(\omega_0-\Omega_s-\Omega) \\ &+\hat{X}(\omega_0-\Omega_s+\Omega)-i\hat{Y}(\omega_0-\Omega_s+\Omega)]/2, \end{aligned}$$

and when $\theta=\pi/2$, the phase quadratures are detected:

$$\begin{aligned} \hat{Y}_+(\Omega) = [&-i\hat{X}(\omega_0+\Omega_s-\Omega)+\hat{Y}(\omega_0+\Omega_s-\Omega)+i\hat{X}(\omega_0 \\ &+\Omega_s+\Omega)+\hat{Y}(\omega_0+\Omega_s+\Omega)]/2, \end{aligned} \tag{18}$$

$$\begin{aligned} \hat{Y}_-(\Omega) = [&-i\hat{X}(\omega_0-\Omega_s-\Omega)+\hat{Y}(\omega_0-\Omega_s-\Omega)+i\hat{X}(\omega_0 \\ &-\Omega_s+\Omega)+\hat{Y}(\omega_0-\Omega_s+\Omega)]/2. \end{aligned}$$

Since from Eq. (15) the upper and lower sidebands are quadrature-amplitude anticorrelated and quadrature-phase correlated, we may determine all of the variances in Eqs. (17) and (18) as

$$\begin{aligned} &\langle \Delta^2[\hat{X}(\omega_0+\Omega_s+\Omega)+\hat{X}(\omega_0-\Omega_s-\Omega)]\rangle/2 = e^{-2r}, \\ &\langle \Delta^2[\hat{X}(\omega_0-\Omega_s+\Omega)+\hat{X}(\omega_0+\Omega_s-\Omega)]\rangle/2 = e^{-2r}, \\ &\langle \Delta^2[\hat{Y}(\omega_0+\Omega_s+\Omega)-\hat{Y}(\omega_0-\Omega_s-\Omega)]\rangle/2 = e^{-2r}, \\ &\langle \Delta^2[\hat{Y}(\omega_0-\Omega_s+\Omega)-\hat{Y}(\omega_0+\Omega_s-\Omega)]\rangle/2 = e^{-2r}, \\ &\Omega \in [0,B/2]. \end{aligned} \tag{19}$$

Therefore, when the two balanced homodyne detectors of the upper and lower sidebands are set at $\theta=0$, the sum output of a hybrid junction (see Fig. 2) is below the SQL [14] with

$$V_+^{hj} = \langle \Delta^2[\hat{X}_+(\Omega)+\hat{X}_-(\Omega)]\rangle/2 = e^{-2r} < 1, \quad \Omega \in [0,B/2]. \tag{20}$$

Similarly, when $\theta=\pi/2$, the difference output of a hybrid junction is below the SQL with

$$V_-^{hj} = \langle \Delta^2[\hat{Y}_+(\Omega)-\hat{Y}_-(\Omega)]\rangle/2 = e^{-2r} < 1, \quad \Omega \in [0,B/2]. \tag{21}$$

These relations therefore demonstrate that the upper- and lower-sideband modes are quadrature-amplitude anticorrelated and quadrature-phase correlated; however, the bandwidth is one-half of that of the squeezed light input. Similarly, if we flip the squeezing of the input squeezed beam (so now $r<0$), then the upper- and lower-sideband modes are quadrature-amplitude correlated with $\langle \Delta^2[\hat{X}_+(\Omega)$

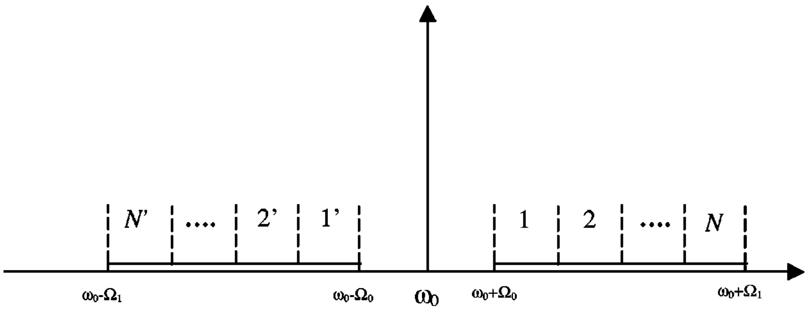

FIG. 3. The spectrum of N entangled EPR pairs produced from a single single-mode squeezed beam. The correlated pairs are labeled j,j'.

$-\hat{X}_-(\Omega)]\rangle/2<1$ and quadrature-phase anticorrelated with $\langle\Delta^2[\hat{Y}_+(\Omega)+\hat{Y}_-(\Omega)]\rangle/2<1$.

We now consider one possible laboratory implementation of the above scheme. We start by noting that the single-mode squeezing generated by a below threshold optical parametric oscillator (OPO) has correlations between its upper and lower sidebands in just the form required. The OPO is described by a moderate-finesse optical resonator containing a χ^2 nonlinear medium pumped by a strong optical field with coherent amplitude α_p and frequency $\omega_p=2\omega_0$. Energy conservation within the rotating-wave approximation requires that the Hamiltonian takes the familiar form

$$H_{\text{int}}=i\hbar g[\alpha_p\hat{a}^\dagger(\omega_0+\Omega)\hat{a}^\dagger(\omega_0-\Omega) - \alpha_p^*\hat{a}(\omega_0+\Omega)\hat{a}(\omega_0-\Omega)], \qquad (22)$$

where g is the coupling strength. Thus the standard finite-bandwidth single-mode squeezing generated by an OPO reveals the EPR-like entanglement between the opposite sidebands. As demonstrated above, these upper and lower sidebands may be spatially separated to produce real EPR pairs, with stronger correlations than usual, though smaller bandwidth. This scheme may be generalized, in principle, so that a single single-mode squeezed beam may be transformed into N EPR entangled pairs by multiple suitably tuned empty optical cavities as shown in Fig. 3. Here, modes $1,1'(2,2';\ldots;N,N')$ form a single pair of an entangled EPR beam with bandwidth $1/2N$ of the input squeezed light. As the transformation involves linear optics, it could be reversed, converting the N frequency-nondegenerate EPR pairs back into single-mode broadband squeezed light with squeezed bandwidth $2N$ times that of any of the EPR pairs.

In conclusion, we have shown the relationship between a single-mode broadband squeezed beam and sideband-entangled EPR pairs. A single broadband beam may be converted into N DSI entangled EPRs, each of which has two spatially frequency-nondegenerate entangled beams; however, the entanglement bandwidth is only $1/2N$ of the squeezed input beam. Single broadband squeezed light has been reported experimentally, emitted by a continuous-wave degenerate optical parametric amplifier with a bandwidth covering the whole frequency range from 1.9 up to 30 MHz [15]. The mature techniques for producing a broadband squeezed beam from a parametric downconverter and employing low-loss empty cavities as well as electronics locking make this scheme feasible experimentally.

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